

# Teaching Dossier

December 2022

Attached is a teaching dossier for Irit Huq-Kuruvilla.

## List of courses taught or assisted

- Fall 2017: Math 1A: Calculus (in-person)
- Fall 2018: Math 54: Linear Algebra and Differential Equations (in-person)
- Summer 2018: Math 110: Linear Algebra (in-person, sample materials Attached)
- Fall 2020: Math 32: Precalculus (online)
- Summer 2022: Math W54: Web-based Linear Algebra and Differential Equations (online)

## Description of Duties and Summary of Attached Documents

For all courses except Math 110, I was responsible for running 2 discussion sections (each 3hr/wk), grading homeworks and exams, and holding office hours. The syllabi and exams for those courses are available on the UC Berkeley Math Department website.

For Math 110, I was essentially the lead instructor for my section of the course. I designed the syllabus, gave lectures, assigned homework, and wrote exams. Homework was graded by a separate grader. The syllabus, midterm exam, and final exam (all written by me), are attached in this document.

Following the course materials are some selected student evaluations. Included are samples of evaluations from Math 110 and Math W54.

# MATH 110: LINEAR ALGEBRA

Summer 2018

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<b>Instructor:</b> Irit Huq-Kuruville	<b>Time:</b> MTWR 2:00 – 4:00
<b>Email:</b> <a href="mailto:irithkmath110@gmail.com">irithkmath110@gmail.com</a>	<b>Place:</b> 136 Barrows.

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## Course Pages:

1. All the relevant information for this course is on the bCourses page. Let me know if you have any trouble accessing this.
2. Assignments will be submitted via Gradescope, the code for accessing the Gradescope page for this course is M7GGB8.

**Office Hours and Contact Information:** My Office Hours will be held in 840 Evans. These are subject to change and a current version will always be on the course homepage on bCourses. Otherwise, you can email me at [irithkmath110@gmail.com](mailto:irithkmath110@gmail.com), which has been set up specifically for this course.

## Textbook:

- Sheldon Axler, *Linear Algebra Done Right*, Springer, 3rd ed., 2015.  
Please let me know if you have any difficulty obtaining the textbook, also note it's not necessary to use the Berkeley edition of this text, if such a thing exists.

**Objectives:** This course is a conceptual treatment of linear algebra with an emphasis on proofs. I'd like this course to not only serve as a way for you to learn the material, but to build and develop skills that will be useful to you along your mathematical journeys.

**Prerequisites:** This course is entirely self-contained, however having previously taken Math 54 will be helpful.

## Course Format:

This course will have a somewhat unconventional style, but I'm making these choices because I'm confident they will better facilitate learning than if I taught this course traditionally.

## Before Class

- The day before each class, you will read a selection from the book, this will substitute for a lecture. The readings for each day are available on the course webpage. Next to the reading, there will be some suggested comprehension exercises from the book.
- After you do your reading for that day, you should email me with an attempt at solving ONE of the comprehension exercises (graded only for completion), and a question or comment relevant to the reading. This question or comment could be: something you didn't understand, something you were confused about but you resolved your confusion (please include how you did this), something from an earlier material that was cleared up by this reading, or something you realized you didn't understand as well as you thought you did. This will be a part of your grade, but you won't necessarily have to do both things each day.

**During class:**

- During class, I'll give a summary of the reading selection, taking your questions and comments into account. After this, I'll give some questions for you to think about individually, and then in groups. Then I'll go over how I'd solve the question. We'll also have weekly review days where I review the content from the week, and take any questions you might have about the material.

**Why?!**

- My goal with this format is to allow you to not only learn linear algebra, but get you more comfortable with reading and learning mathematics on your own. If you're in any job that requires math, you'll need to be able to learn new results and techniques on your own, this is a skill unto itself and may not be one you'll get practice with if you're used to lecture-based courses.
- In addition, being able to follow complicated arguments by yourself and read texts on your own are skills that will serve you well in future upper-division math classes.
- These kinds of methods have also empirically been shown to be better for students.

**Homework & Exams**

- There will be weekly assigned homework, available on the course website, Your homeworks are due on Thursdays, submitted electronically to Gradescope. (Let me know if you have an issue submitting assignments in this way). NOTE: You must submit your homework as separate images, one for each problem.
- There will be no quizzes, but there will be a midterm and a final exam, both will take place in class.

**Grading Policy:**

- Homework: 15% of your grade. There will be 7 HW assignments, of which your lowest 2 will be dropped.
- Midterm: 30% of your grade. Your midterm score can be replaced by your final exam score if that would improve your grade.
- Final exam: 45% of your grade. Both the midterm and the final exam will be curved if necessary. Curves can only improve your grade, I will never curve down.
- Reading Responses: 10% of your grade. Your score for these will be calculated out of 40 points. Each day, you can earn 2 points (1 for an attempt at a comprehension question, and 1 for a question or comment about the material). Since there are 30 class days, you do not have to do both things every day.
- Regrades: All items, except for the final, will be published on Gradescope, you are welcome to submit regrade requests via Gradescope until a week after the grade for an assignment has been published. It is possible for a regrade request to result in lowering your grade, we obviously will not try to make this happen, but you should only submit requests for where you feel what is given to you on the rubric does not reflect the work you have done.

**Important Dates:**

Midterm .....Monday, July 16, 2018  
 Final Exam ..... Thursday, August 9, 2018

**DSP:**

- If you have DSP accommodations please send me your letter as possible, and let me know what accommodations you'd like, so I can make sure everything is set up for you.

**Academic Honesty:** Students are expected to adhere to UC Berkeley's Honor Code. Lack of knowledge of the academic honesty policy is not a reasonable explanation for a violation. Violating the Honor Code can potentially result in failure of the course or even expulsion from the University.

# Math 110 Midterm

Name: \_\_\_\_\_

Student ID Number: \_\_\_\_\_

Answer the questions in the spaces provided. Please do not insert additional sheets of paper into the exam, or remove existing pages. There are extra blank sheets provided at the end of the exam, and each page has a blank back. If you are writing any solutions on these extra sheets, or on the back of one of the pages, indicate that you have done so on the page for the relevant problem.

You have 2 hours (2:10 to 4:10) to complete the exam, and you are allowed no calculators, notebooks, cheat sheets, or other aids. Any violation of the University's honor code will result in failure of the exam.

The questions in each section are meant to be of varying difficulty, don't spend all your time on one section of the test, if you get stuck, I suggest you try moving onto the next section, and returning later to the previous section if you have time.

## Index of notation:

- $F$ : A field, either the real numbers ( $\mathbb{R}$ ), or the complex numbers ( $\mathbb{C}$ ).
- $\mathcal{L}(V, W)$ : The space of linear transformations between  $V$  and  $W$ .
- $\mathcal{L}(V)$ : The space of linear operators from  $V$  to itself.
- $V'$  (where  $V$  is a vector space): The dual space of  $V$
- $T'$  (where  $T$  is a linear transformation) : The dual transformation  $T$
- $f'$  (where  $f$  is a differentiable function): The derivative of  $f$ .
- $P(F)$ : The vector space of polynomials over  $F$
- $P_m(F)$ : The vector space of polynomials over  $F$  of degree at most  $m$ .
- $U^0$ : The annihilator of  $U$
- $\oplus$ : Direct sum.
- $F^n$ : The vector space of lists of elements of  $F$  of length  $n$ .
- $F^S$  ( $S$  a set): The vector space of functions from  $S$  to  $F$ .
- $F^\infty$ : The vector space of "infinite lists" of elements of  $F$ .
- $I$ : The identity in  $\mathcal{L}(V)$ .

**Note** In general, vector spaces of the form  $F^n, F^S, P(F)$  etc. are assumed to be defined over  $F$  unless said otherwise. If you aren't sure what field a vector space is supposed to be defined over, please ask.

## Part I: True/False

For this part of the exam, circle your answers to each question. No justification is necessary. You will receive 1 point per correct answer, 0 points per blank answer, and -1 points for any incorrect answers. However your score for this part cannot be below 0.

1. True    False    A system of  $n$  linear equations in  $n$  variables always has a unique solution
2. True    False    The set  $\mathbb{R}$  with the usual addition operation, and scalar multiplication by  $\mathbb{C}$  with  $(a + bi)v = av$  is a vector space over  $\mathbb{C}$ .
3. True    False    If  $V = \text{span}(v_1) \oplus \text{span}(v_2)$ ,  $v_1, v_2$  are linearly independent.
4. True    False    An operator in  $\mathcal{L}(V)$  is injective if and only if it's surjective.
5. True    False    Let  $T \in \mathcal{L}(V, W)$ , with  $V$  and  $W$  finite dimensional vector spaces choosing a basis for  $V$  uniquely specifies a matrix for  $T$ .
6. True    False    Every vector space over  $F$  of dimension  $d$ , where  $d$  is a finite nonnegative integer, is isomorphic to the space  $F^d$
7. True    False    There is a unique linear transformation  $P_3(\mathbb{R}) \rightarrow F^\infty$  sending  $x$  to  $(1, 0, 0, \dots)$ ,  $x^2$  to  $(1, 1, 1, 1, \dots)$ , and  $x^3$  to  $(5, 6, 7, 8, 9, \dots)$ .
8. True    False    Any spanning set of polynomials of different degrees forms a basis of  $P_m(\mathbb{R})$
9. True    False    Any basis of  $P_m(\mathbb{R})$  must consist of polynomials with different degrees.
10. True    False    There is a transformation in  $\mathcal{L}(\mathbb{R}^5, \mathbb{R}^3)$ , whose null space is  $\{(a, b, c, d, e) \in \mathbb{R}^5 : (a + b = 0)\}$
11. True    False    The function  $T$  from  $P(\mathbb{C})$  to itself, defined by  $Tp = \bar{p}$  is linear over  $\mathbb{C}$ .
12. True    False    If  $U \subset W$  are subspaces of any vector space  $V$ , then  $U^0 \subset W^0$
13. True    False    Given operators  $T, S \in \mathcal{L}(V)$ , with  $TS = I$ , then  $T$  is invertible.
14. True    False     $T \in \mathcal{L}(V, W)$  is injective iff  $T'$  is surjective
15. True    False     $T \in \mathcal{L}(V, W)$  is surjective iff  $T'$  is injective

## Part II: Calculations

For this part of the exam. Calculate the dimension of the following vector spaces  $V$ , and write your final answer on the dotted line next to the question. No justification is required, if the vector space is infinite-dimensional, just write  $\infty$ . Each correct answer is worth 1 points.

1.  $V = P_3(\mathbb{R})$ . -----
2.  $V = \text{Null}(T)$  where  $T$  in  $\mathcal{L}(\mathbb{R}^2, \mathbb{R}^2)$  is represented in the standard basis by  $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ . -----
3.  $V = \text{Null}(T')$ , where  $T \in \mathcal{L}(\mathbb{R}^7, \mathbb{R}^2)$ , with  $\dim(\text{Null}(T)) = 5$ . -----
4.  $V = W'/U^0$ , where  $W$  is a vector space of dimension 8, and  $U$  a subspace of  $W$  of dimension 6. Calculate  $\dim(V'/U^0)$ . -----
5.  $V = \text{Null}(T)$ , where  $W$  is the subspace of  $\mathbb{R}^{\mathbb{R}}$  consisting of infinitely differentiable functions and  $T$  is the operator on  $W$  sending  $f$  to  $f''$ . -----

### Part III: Examples

For this part of the exam. Provide examples for the stated phenomena, you can describe them in any way you'd like, and no justification is required.

1. (2 points) Find an example of a vector space  $V$  and two operators  $S, T \in \mathcal{L}(V)$  such that  $ST \neq TS$ .
2. (2 points) Find an example of a  $T \in \mathcal{L}(\mathbb{R}^2, \mathbb{R}^2)$  such that  $\text{Null}(T) = \text{Range}(T)$ .
3. (2 points) Find an example of an operator in  $\mathcal{L}(\mathbb{R}^\infty)$ , such that  $\text{Null}(V)$  is finite dimensional.
4. (2 points) Let  $V = \mathbb{R}^2$ , let  $f$  be the element of  $V'$  that sends  $(x, y)$  to  $x + y$ , let  $g$  be the element of  $V'$  that sends  $(x, y)$  to  $3x + 5y$ . Find a basis  $v_1, v_2$  for  $V$  such that  $f, g$  is its dual basis.
5. (2 points) Find an example of a polynomial  $p$  in  $P(\mathbb{R})$  that cannot be expressed as a product of two polynomials in  $P(\mathbb{R})$  of smaller degree, but can be expressed this as a product of two polynomials in  $P(\mathbb{C})$  of smaller degree.



### Part III: Proof

For this part of the exam, answer the questions below as completely as possible, with proofs. You are welcome to cite any results from class or from the textbook that you wish.

1. (10 points)

Let  $V, W, X$  be vector spaces over  $F$ . Let  $S$  be in  $\mathcal{L}(V, W)$ .  $T$  be in  $\mathcal{L}(W, X)$ . Show that the set:

$$\{(v, Sv, Tv) \in V \times W \times X\}$$

is a subspace of  $V \times W \times X$ .

2. 10 points Let  $v_1, v_2, v_3$  be linearly independent vectors in a vector space  $V$  over  $\mathbb{R}$ . For which values of  $k \in \mathbb{R}$  are the vectors  $v_2 - v_1, kv_3 - v_2, v_1 - v_3$  linearly independent.

3. (10 points) An *idempotent* operator on a vector space  $V$  is an operator  $T \in \mathcal{L}(V)$  such that  $T^2 = T$ . Examples of idempotent operators include the identity (written as  $I$ ) and the 0 transformation. (Here  $T^2$  denotes the composition of  $T$  with itself.)

(a) (4 points) Prove that if  $T$  is idempotent, then  $I - T$  is also idempotent.

(b) (6 points) Prove that if  $T$  is an idempotent operator that is NOT the identity,  $\dim(\text{Null}(T)) > 0$ .

# Math 110 Final Exam

Name: \_\_\_\_\_

Student ID Number: \_\_\_\_\_

Answer the questions in the spaces provided. Please do not insert additional sheets of paper into the exam, or remove existing pages. There are extra blank sheets provided at the end of the exam, and each page has a blank back. If you are writing any solutions on these extra sheets, or on the back of one of the pages, indicate that you have done so on the page for the relevant problem.

You have 2 hours (2:10 to 4:10) to complete the exam, and you are allowed no calculators, notebooks, cheat sheets, or other aids. Any violation of the University's honor code will result in failure of the exam.

The questions in each section are meant to be of varying difficulty, don't spend all your time on one section of the test, if you get stuck, I suggest you try moving onto the next section, and returning later to the previous section if you have time.

This time there is a bonus 4<sup>th</sup> proof question. It's a bit harder than the others and not worth quite as many points.

## Index of Notation:

- $F$ : A field, either the real numbers ( $\mathbb{R}$ ), or the complex numbers ( $\mathbb{C}$ ).
- $\mathcal{L}(V, W)$ : The space of linear transformations between  $V$  and  $W$ .
- $\mathcal{L}(V)$ : The space of linear operators from  $V$  to itself.
- $V'$  (where  $V$  is a vector space): The dual space of  $V$
- $T'$  (where  $T$  is a linear transformation) : The dual transformation  $T$
- $f'$  (where  $f$  is a differentiable function): The derivative of  $f$ .
- $P(F)$ : The vector space of polynomials over  $F$
- $P_m(F)$ : The vector space of polynomials over  $F$  of degree at most  $m$ .
- $U^0$ : The annihilator of  $U$
- $\oplus$ : Direct sum.
- $V \times U$ : The product of the vector spaces  $V$  and  $W$ .
- $V/U$ : The quotient of  $V$  by a subspace  $U$ .
- $T/U$ : The quotient operator in  $\mathcal{L}(V/U)$  of an operator  $T \in \mathcal{L}(V)$ , where  $U$  is a  $T$ -invariant subspace.
- $T|_U$ : The restriction operator in  $\mathcal{L}(U)$  of an operator  $T \in \mathcal{L}(V)$ , where  $U$  is a  $T$ -invariant subspace.
- $F^n$ : The vector space of lists of elements of  $F$  of length  $n$ .
- $F^S$  ( $S$  a set): The vector space of functions from  $S$  to  $F$ .
- $F^\infty$ : The vector space of “infinite lists” of elements of  $F$ .
- $I$ : The identity in  $\mathcal{L}(V)$ .
- $E(\lambda, T)$ : The eigenspace of  $T$  corresponding to the eigenvalue  $\lambda$ .
- $G(\lambda, T)$ : The generalized eigenspace of  $T$  corresponding to the eigenvalue  $\lambda$ .
- $u \cdot v$ : The dot product of  $u, v \in F^n$ .
- “inner product space”: A vector space with an inner product.
- $\langle u, v \rangle$ : If  $u, v$  are vectors in an inner product space  $V$ , this denotes the inner product of  $u$  and  $v$ .
- $\|v\|$ : The norm of the vector  $v$  (This is also  $\sqrt{\langle v, v \rangle}$ ).
- $T^*$ : The adjoint of a linear transformation  $T$
- $U^\perp$ : The orthogonal complement of a subspace  $U$ .
- $P_U$ : The operator in  $\mathcal{L}(V)$  that is the orthogonal projection onto a subspace  $U$ .

**Note** In general, vector spaces of the form  $F^n, F^S, P(F)$  etc. are assumed to be defined over the field  $F$  unless said otherwise. If you aren't sure what field a vector space is supposed to be defined over, please ask.

**Note 2:** If any reference is made to an inner product on  $F^n$ , that will be the dot product unless I've written otherwise.

## Part I: True/False

For this part of the exam, fill in the circle corresponding to your answer **COMPLETELY**. Do not put a check or  $\times$  mark. If you need to change your answer, do your best to erase clearly. No justification is necessary. You will receive 1 point per correct answer, 0 points per blank answer, and  $-1/2$  points for any incorrect answers. However your total score for this part cannot be below 0.

1.  True      Let  $V$  be finite dimensional inner product space and  $U$  a subspace of  $V$ , then  $V = U \oplus U^\perp$   
 False
2.  True      If  $v, w$  are not eigenvectors of an operator  $T$ , then  $v + w$  is not an eigenvector of  $T$ .  
 False
3.  True      If  $V$  is an inner product space, then  $\langle v, u + w \rangle = \langle v, u \rangle + \langle v, w \rangle$  for all  $u, v, w \in V$   
 False
4.  True      Let  $T \in \mathcal{L}(V)$ , then  $\text{Null}(T^5 + 6T)$  is invariant under  $T$ .  
 False
5.  True      If  $T \in \mathcal{L}(V)$  is a positive operator, then  $T$  has a unique square root.  
 False
6.  True      If  $T \in \mathcal{L}(V, W)$ , the quotient operator  $T/\text{Null}(T) \in \mathcal{L}(V/\text{Null}(T), W)$  is injective.  
 False
7.  True      If  $U$  is a subspace of an inner product space  $V$ , then  $P_U^2 = P_U$ .  
 False
8.  True       $\begin{pmatrix} i \\ 2 + 3i \end{pmatrix} \cdot \begin{pmatrix} 1 + i \\ 2i \end{pmatrix} = 5i - 7$   
 False
9.  True      Every  $T \in \mathcal{L}(\mathbb{C})$  is self-adjoint.  
 False
10.  True      For  $V$  a finite dimensional vector space, if  $\langle Tv, v \rangle = 0$  for all  $v \in V$ , then  $T = 0$ .  
 False
11.  True      Let  $T$  be an operator in  $\mathcal{L}(\mathbb{C}^6)$ , if  $T^{10}(1 + i, i, 6, 7, 3 + 5i, 2) = \vec{0}$ , then  $T^6(1 + i, i, 6, 7, 3 + 5i, 2) = \vec{0}$   
 False  
(Note: here I'm writing the vectors in  $\mathbb{C}^6$  horizontally to save space)
12.  True      The operator  $D \in \mathcal{L}(P(\mathbb{R}))$  sending a polynomial to its derivative is nilpotent.  
 False
13.  True      Let  $T \in \mathcal{L}(V)$ , where  $V$  is a finite dimensional inner product space be such that  $T^2 = -I$ . Then  $T$  is not self-adjoint.  
 False
14.  True      If  $T \in \mathcal{L}(\mathbb{C}^n)$  is diagonalizable, then there exists an orthonormal basis of  $\mathbb{C}^n$  in which  $\mathcal{M}(T)$  is diagonal.  
 False
15.  True      There exists a polynomial  $q \in P_{10}(\mathbb{R})$  such that for any polynomial  $p \in P_{10}(\mathbb{R})$ ,  
 False  
 $p(5) = \int_{-1}^1 p(x)q(x)dx$

## Part II: Calculations

For this part of the exam. Calculate the following numbers. Each correct answer is worth 1 point. Write your final answers in the blank spaces at the beginning of each question. Do your best to write clearly.

- \_\_\_\_\_ Find  $\dim V$  where  $V = (U/W) \times (X/Y)$  and  $\dim U = 6$ ,  $\dim W = 4$ ,  $\dim X = 7$ ,  $\dim Y = 2$ .
- \_\_\_\_\_ Find  $\dim V$  where  $V = \mathcal{L}(\mathbb{R}^3, \mathbb{C}^3)$ , where both  $\mathbb{R}^3$  and  $\mathbb{C}^3$  are considered as vector spaces over  $F = \mathbb{R}$ .
- \_\_\_\_\_ Calculate the norm of the vector  $5x^2 + 3$  in the real vector space  $P_2(\mathbb{R})$ , with the inner product given by  $\langle p, q \rangle = \int_{-1}^1 pq dx$
- \_\_\_\_\_ Find  $\dim V$  where  $V = E(2, T)$ , where  $T$  is given in the standard basis by the following matrix:  
$$\begin{pmatrix} 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$
- \_\_\_\_\_ Find the maximum number of Jordan blocks of size 3 of an operator  $T$  in  $\mathcal{L}(\mathbb{C}^9)$  whose characteristic polynomial is  $(x - 5)^7(x - 3)(x - 2)$  and  $\dim E(5, T) = 3$ .

### Part III: Examples

For this part of the exam. Provide examples for the stated phenomena, you can describe them in any way you'd like, and no justification is required. If no example exists, write "impossible".

1. (2 points) Find an example of an operator  $T \in \mathcal{L}(\mathbb{C}^5)$  whose characteristic polynomial is  $(x - 3)^2(x - 4)^2(x - 5)$ , but whose minimal polynomial is  $(x - 3)(x - 4)^2(x - 5)$ . Express your answer (if one exists) as a matrix in Jordan form.
2. (2 points) Let  $V$  be the subspace of  $\mathbb{R}^{[-1,1]}$  consisting of continuous functions. Define the inner product on  $V$  in the following way:  $\langle f, g \rangle = \int_{-1}^1 fg dx$ . Find an example (if one exists) of two nonzero vectors in  $V$  that are orthogonal to each other.
3. (2 points) Find an example of a self-adjoint operator in  $\mathcal{L}(\mathbb{C}^2)$  whose characteristic polynomial is  $x^2 + 1$ . Express your answer (if one exists) as a matrix in Jordan form.
4. (2 points) Find an example of a **nonzero** nilpotent operator  $N$  in  $\mathcal{L}(\mathbb{C}^5)$  such that there **doesn't** exist a basis of  $\mathbb{C}^5$  of the form  $N^4v, N^3v, N^2v, Nv, v$ . Express your answer (if one exists) as a matrix in Jordan form.
5. (2 points) Find an example (if one exists) of a normal operator in a complex vector space that is not self-adjoint. (Please write which vector space and inner product you are considering).



## Part IV: Proof

For this part of the exam, answer the questions below as completely as possible, with proofs. You are welcome to cite any results that you wish from class, homework, or from chapters 1-8 of the textbook.

1. (10 points)

Let  $V$  be a real inner product space. Let  $u, v$  be vectors in  $V$ . Prove that  $\langle u, v \rangle = \frac{\|u+v\|^2 - \|u-v\|^2}{4}$ .

2. (10 points)

Two operators  $A, B \in \mathcal{L}(V)$  are said to *commute* if  $AB = BA$ .

(a) (5 points) Prove that if  $A$  and  $B$  commute, the eigenspaces  $E(\lambda, B)$  of  $B$  are  $A$ -invariant.

(b) (5 points)

**Theorem.** *If  $T \in \mathcal{L}(V)$  is diagonalizable, and  $U$  is a  $T$ -invariant subspace of  $V$ , then  $T|_U$  is diagonalizable.*

Use the above theorem and part a) to conclude that if  $A, B \in \mathcal{L}(V)$  commute and they are both diagonalizable, then  $A$  and  $B$  can be diagonalized in the same basis.

3. (10 points)

Let  $V$  be a complex finite dimensional inner product space. An operator  $T \in \mathcal{L}(V)$  is called *skew-Hermitian* if  $T^* = -T$ .

(a) (5 points) Prove that any skew-Hermitian operator is diagonalizable.

(b) (5 points) Prove that the eigenvalues of a skew-Hermitian operator must be purely imaginary (this means that they are of the form  $bi$  for some real number  $b$ ).

4. **Bonus:** (5 points)

Let  $V$  be a finite dimensional complex inner product space. Let  $A, B \in \mathcal{L}(V)$  be normal operators such that  $\text{Range}(A)$  is orthogonal to  $\text{Range}(B)$ . (this means that any vector in  $\text{Range}(A)$  is orthogonal to any vector in  $\text{Range}(B)$ , and vice versa, equivalently  $\text{Range}(A) \subset \text{Range}(B)^\perp$ ). Prove that  $A + B$  is normal.



## Huq-Kuruvilla, Irit (MATH 110 LEC 004 LINEAR ALGEBRA) - Su 2018 (Instructor Version)

Project Title: **Summer 2018 Evaluations**

Project Audience: **32**

Responses Received: **25**

Response Ratio: **78.12%**

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### Subject Details

Name	MATH 110 LEC 004 LINEAR ALGEBRA
DEPT_NAME	MATH
DEPT_FORM	MATH
EVALUATION_TYPE	F
First Name	Irit
Last Name	Huq-Kuruvilla

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Creation Date: **Wed, Sep 05, 2018**

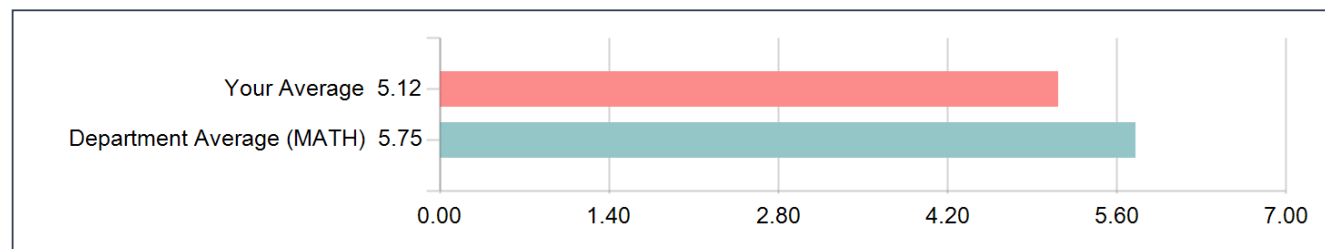


**FOR YOUR INFORMATION:** Please note that "Department Average" for each rating question is calculated using all sections in your department. This may include both Faculty and GSIs depending on whether the department has selected a question item to be used for both.

## RATING QUESTIONS (QUANTITATIVE)

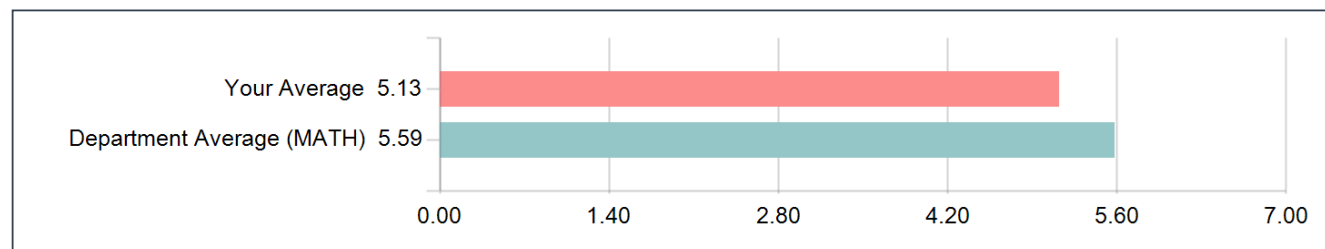
*UNIVERSITY WIDE QUESTIONS: The quantitative items in this section are asked across all courses at Berkeley.*

**Considering both the limitations and possibilities of the subject matter and the course, how would you rate the overall effectiveness of this instructor?**



Options	Count	Percentage
1-Not at all Effective	0	0.00%
2	1	4.00%
3	3	12.00%
4-Moderately Effective	5	20.00%
5	2	8.00%
6	11	44.00%
7-Extremely Effective	3	12.00%
Statistics		Value
Response Count		25
Mean		5.12
Median		6.00
Standard Deviation		1.42

**Considering both the limitations and possibilities of the subject matter and the course, how would you rate the overall effectiveness of this course?**

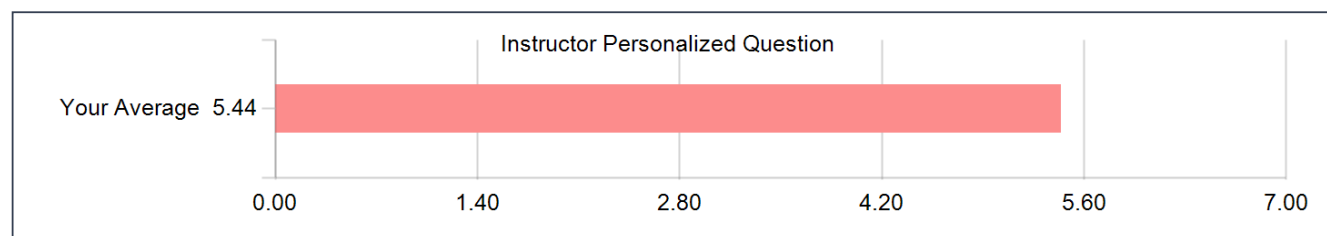


Options	Count	Percentage
1-Not at all Effective	0	0.00%
2	0	0.00%
3	1	4.17%
4-Moderately Effective	7	29.17%
5	7	29.17%
6	6	25.00%
7-Extremely Effective	3	12.50%
Statistics		Value
Response Count		24
Mean		5.13
Median		5.00
Standard Deviation		1.12

*DEPARTMENT PROVIDED RATING QUESTIONS: Questions in this section were selected by your department for inclusion on this evaluation.*

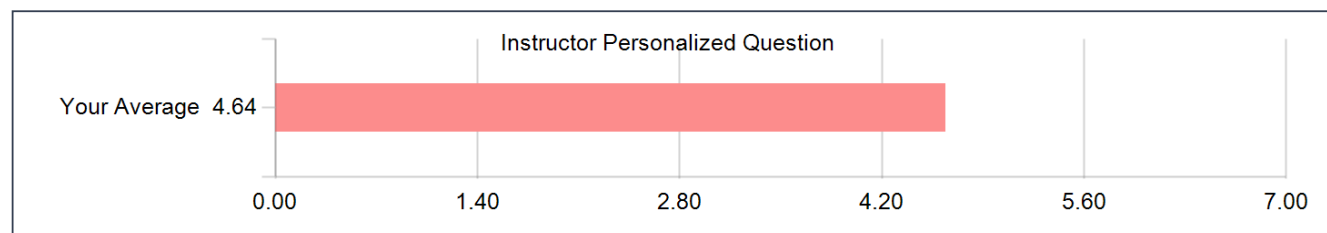
*INSTRUCTOR PROVIDED QUESTIONS (CUSTOM): If any rating questions appear in this section, they were created by you. If blank, you did not add any custom items to your evaluation. These are viewable only by you and not accessible by other report viewers in your department.*

#### To what extent do you feel this class improved your ability to read and understand mathematical proofs?



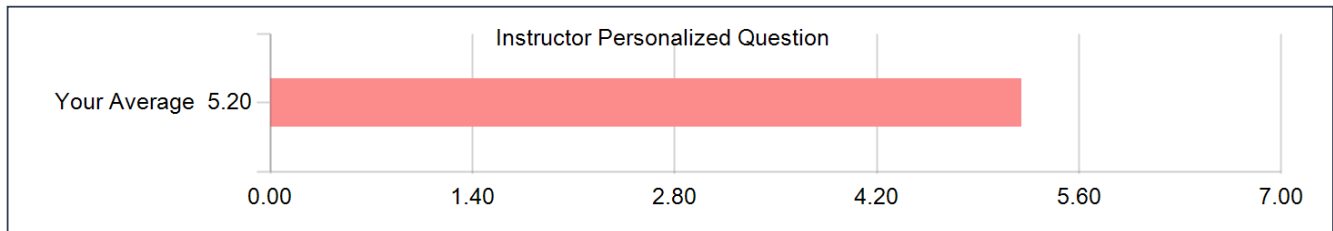
Options	Count	Percentage
1-Not at all	0	0.00%
2	0	0.00%
3	1	4.00%
4-Somewhat	4	16.00%
5	6	24.00%
6	11	44.00%
7-Very	3	12.00%
Statistics		Value
Response Count		25
Mean		5.44
Median		6.00
Standard Deviation		1.04

#### To what extent do you feel this class improved your ability to write mathematical proofs?



Options	Count	Percentage
1-Not at all	0	0.00%
2	2	8.00%
3	2	8.00%
4-Somewhat	7	28.00%
5	8	32.00%
6	4	16.00%
7-Very	2	8.00%
Statistics		Value
Response Count		25
Mean		4.64
Median		5.00
Standard Deviation		1.32

**To what extent do you feel the course was preferable to a standard lecture course?**



Options	Count	Percentage
1-Not at all	0	0.00%
2	1	4.00%
3	1	4.00%
4-Somewhat	5	20.00%
5	9	36.00%
6	3	12.00%
7-Very	6	24.00%
Statistics		Value
Response Count		25
Mean		5.20
Median		5.00
Standard Deviation		1.35

## OPEN ENDED QUESTIONS (QUALITATIVE)

*DEPARTMENT PROVIDED QUESTIONS: Questions in this section were selected by your department for inclusion on this evaluation.*



**Comments on any other relevant aspects of the course such as content, text, how it could be improved, advice to people who have to take it, etc.**

- o On the first few days of the course we should have gone over proofs because it took me a while to figure them out.

**Considering both the limitations and possibilities of the subject matter and the course, how would you rate the overall effectiveness of this course?**

- o 5

**Your Major? (List All)**

- o Physics

**Subject Name: MATH 110 LEC 004 LINEAR ALGEBRA**

**Secondary Subject Name: Irit Huq-Kuruville**

**What are the instructor's strengths? (i.e., preparation and organization of lectures, content, boardwork, examples, clarity, willingness to answer questions, attitude toward students, office hours, homework, exams, grading).**

- o clear explanation. easy to understand.

**What are the instructor's weaknesses? How could the instructor improve his/her teaching?**

- o spending too much time discussing. can be faster.

**Considering both the limitations and possibilities of the subject matter and the course, how would you rate the overall effectiveness of this instructor?**

- o 6

**To what extent do you feel this class improved your ability to read and understand mathematical proofs?**

- o 6

**To what extent do you feel this class improved your ability to write mathematical proofs?**

- o 4-Somewhat

**To what extent do you feel the course was preferable to a standard lecture course?**

- o 6

**What (if anything) do you think you've gotten out of the course that you'll use in future courses or later in your life?**

- o study by myself

**Considering both the limitations and possibilities of the subject matter and the course, how would you rate the overall effectiveness of this course?**

- o 7-Extremely Effective

**Your Major? (List All)**

- o Mathematics and economics

**Subject Name: MATH 110 LEC 004 LINEAR ALGEBRA**

**Secondary Subject Name: Irit Huq-Kuruville**

**What are the instructor's strengths? (i.e., preparation and organization of lectures, content, boardwork, examples, clarity, willingness to answer questions, attitude toward students, office hours, homework, exams, grading).**

- o He is always prepared and the lectures are always very orderly. He has a very open attitude towards students and answers any question asked completely. He also goes through lots of examples in the latter half of the class.

**What are the instructor's weaknesses? How could the instructor improve his/her teaching?**

- o I don't see any glaring weaknesses in the instruction.

**Considering both the limitations and possibilities of the subject matter and the course, how would you rate the overall effectiveness of this instructor?**

- o 6

**To what extent do you feel this class improved your ability to read and understand mathematical proofs?**

- o 7-Very

**To what extent do you feel this class improved your ability to write mathematical proofs?**

- o 6

**To what extent do you feel the course was preferable to a standard lecture course?**

- o 5

**What (if anything) do you think you've gotten out of the course that you'll use in future courses or later in your life?**

- o I've learned many skills in both reading and writing proofs from this course which I will likely use a lot in the future.



# Huq-Kuruvilla, Irit (MATH W54 WBD 101 LIN ALG & DIFF EQNS) - Summer 2022 (Instructor Version)

Project Title: **Summer 2022 Evaluations**

Courses Audience: **44**

Responses Received: **34**

Response Ratio: **77.27%**

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Subject Details	
Name	MATH W54 WBD 101 LIN ALG & DIFF EQNS
DEPT_NAME	MATH
DEPT_FORM	MATH
EVALUATION_TYPE	G
First Name	Irit
Last Name	Huq-Kuruvilla

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Creation Date: **Friday, September 16, 2022**

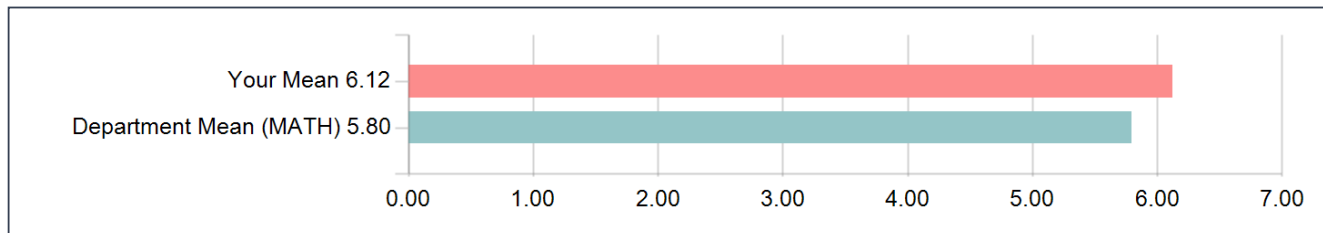


**FOR YOUR INFORMATION:** Please note that "Department Mean" for each rating question is calculated using all sections in your department. This may include both Faculty and GSIs depending on whether the department has selected a question item to be used for both.

**UNIVERSITY WIDE QUESTIONS (QUANTITATIVE/RATING):**

*The items in this section are asked across all courses at Berkeley.*

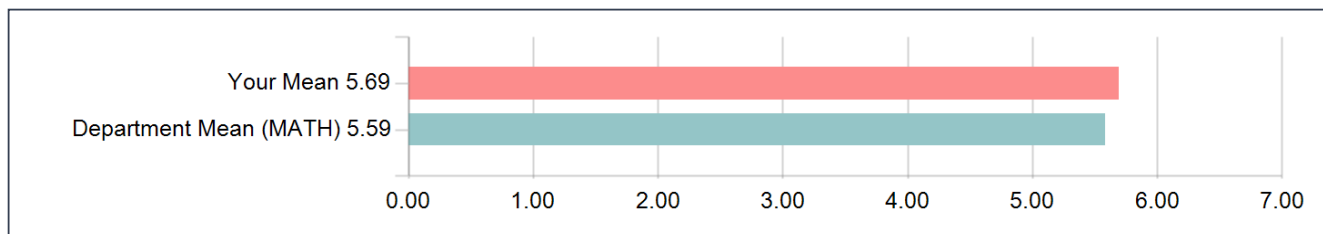
Considering both the limitations and possibilities of the subject matter and the course, how would you rate the overall effectiveness of this graduate student instructor?



Considering both the limitations and possibilities of the subject matter and the course, how would you rate the overall effectiveness of this graduate student instructor?

Options	Count	Percentage
1-Not at all Effective	0	0.00%
2	0	0.00%
3	1	4.00%
4-Moderately Effective	2	8.00%
5	2	8.00%
6	8	32.00%
7-Extremely Effective	12	48.00%
Statistics		Value
Response Count		25
Mean		6.12
Median		6.00
Standard Deviation		1.13

Considering both the limitations and possibilities of the subject matter and the course, how would you rate the overall effectiveness of this course?



Considering both the limitations and possibilities of the subject matter and the course, how would you rate the overall effectiveness of this course?

Options	Count	Percentage
1-Not at all Effective	1	3.85%
2	0	0.00%
3	2	7.69%
4-Moderately Effective	3	11.54%
5	3	11.54%
6	5	19.23%
7-Extremely Effective	12	46.15%
Statistics		Value
Response Count		26
Mean		5.69
Median		6.00
Standard Deviation		1.64

## DEPARTMENT PROVIDED INSTRUCTOR QUESTIONS:

*Items in this section were selected by MATH for inclusion on this evaluation.*

**What are the Graduate Student Instructor's strengths (i.e., preparation and organization, willingness to answer questions, attitude toward students, availability and usefulness of office hours)?**

Comments
I liked how active they were on the Ed, how there were multiple discussion sections available and they announced what they were going over before the section started.
n/a
quick responding, organized
willingness to answer questions
super available/accommodating!
willingness to answer questions
preparation and organization
I did not attend the discussion sections!
Very organized, thorough, and responsive! Kishan created a 40 page study guide for us!
great
Several discussion was useful
This instructor is generally well prepared to answer questions brought by students who attended their sessions. He uses various examples to help students strengthen their knowledge and is always helpful in the ED forum with answering questions asked by students.
I, unfortunately, had a time conflict during the discussion section so I never had a chance to be taught by Irit. However, based on their Ed discussion posts they seemed helpful.
I did not have the chance to go to their discussion section, so I am not sure.
patient
Preparation and Organization
good
-

**What suggestions for improvement do you have for the Graduate Student Instructor? How could the instructor improve their teaching?**

Comments
n/a
No suggestions
n/a
preparation and organization
be more interactive but obviously it's hard to do remotely
I did not attend the discussion sections!
None, honestly they were all amazing!
N/A
Nothing to be suggested, great job being done by him.
I, unfortunately, had a time conflict during the discussion section so I never had a chance to be taught by Irit.
I did not have the chance to go to their discussion section, so I am not sure.
n/a
N/A
good
-

**Any other comments about this graduate student instructor?**

Comments
n/a
helpful with technical difficulties as an abroad student
None
n/a
no
n/a
n/a
N/A
Thank you, you helped me a lot during this summer, along with other instructors.
No
n/a
N/A
good
-

**DEPARTMENT PROVIDED COURSE QUESTIONS:**

*Items in this section were selected by MATH for inclusion on this evaluation.*

**Any other comments about this discussion section?**

Comments
n.a
None
n/a
no
Organized and thorough.
The time for the discussion section is flexible, and the contents covered in those sessions are useful.
n/a
good
-

## DEPARTMENT PROVIDED STUDENT INFORMATION QUESTIONS

*Items in this section were selected by MATH for inclusion on this evaluation.*

### Your Major? (List All)

Comments
Econ and Media Studies Double Major and Minor in Data Science
Chemical Engineering
Data Science
Economics Data Science
Data Science
mechanical engineering
Data Science
Intended Major: Mathematics
No Major / Summer
Mechanical Engineering Engineering Math and Statistics
psychology, data science
Cognitive Science, Computer Science
Econ
Intended Physics, Intended Computer Science
Business Administration
intended stats
Mechanical Engineering
Data Science and MCB
Math
CS+Stat
Undeclared, willing to declare as data science major.
I am still a high school junior student at Chaminade College Preparatory. Still, after a summer of studying in the Berkeley summer program, I am determined to study electrical engineering in the future.
Political Science, Economics
Business Administration Political Economy
Chemical Engineering
data science and economics
Economics, Computer Science
good
chemical engineering